CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 16 December 2025
Time: 13.30 – 16.30
Questions: 4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:				
Question	1	2	3	4
a b c d	4 5 6 6	5 6 6 8	4 5 6 6	7 7
Total	21	25	21	14

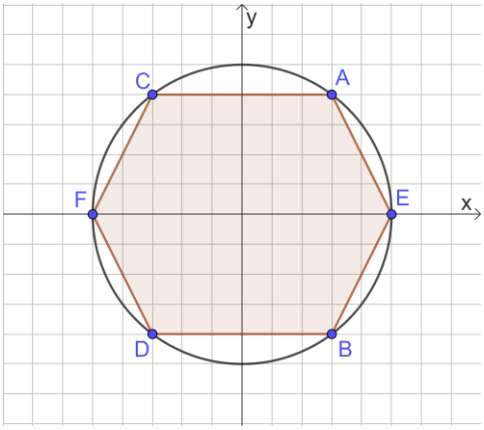
Grade $= \frac{\text{total points scored}}{9} + 1$

You will pass the exam if your grade is at least 5.5 .

Question 1 – A family of hexagons in a circle

Take a new answer sheet for every question!

Given is the circle c with equation $x^2 + y^2 = 25$.
For $0 < p < 5$, six points are given on this circle:
Points A and B are the intersections of the vertical line $x = p$ and circle c.
Points C and D are the intersections of the vertical line $x = -p$ and circle c.
Points E and F are the intersections of the *x*-axis and circle c.
These points are the corners of a hexagon H_p.
The hexagon H_3 is shown in the figure below as an example.



4pt a For the hexagon in the figure (so for $p = 3$), compute algebraically $\angle E$, that is the angle between sides EA and EB in point E.
Give your answer in degrees, rounded to one digit behind the decimal point.

5pt b For $p = 1\frac{2}{5}$, use an exact computation to find an equation for the tangent line to circle c in point A.

For all values of p with $0 < p < 5$, the area of hexagon H_p can be computed with the formula $A(p) = (2p + 10) \cdot \sqrt{25 - p^2}$.

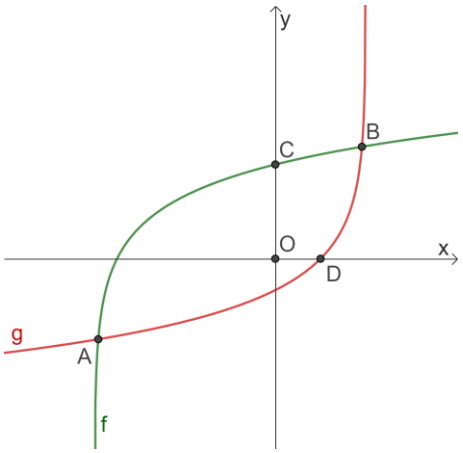
6pt c Show this.

6pt d Compute exactly the value of p for which the area of the hexagon H_p is maximal.

Question 2 – Two logarithmic functions and a circle

Take a new answer sheet for every question!

The functions f and g are given by $f(x) = \ln(2x + 8)$ and $g(x) = -\ln(2 - x)$.
In the figure below, the graphs of these functions are shown, together with the following five points:
Point *O* is the origin $(0,0)$.
Points *A* and *B* are the intersections of the graphs of f and g with $x_A < x_B$.
Point *C* is the intersection of the graph of f and the *y*-axis.
Point *D* is the intersection of the graph of g and the *x*-axis.



5pt a Compute exactly the *x*-coordinates of the points *A* and *B*.

The vertical line $x = p$ with $x_A < p < x_B$ intersects the graph of f in the point F_p and the graph of g in the point G_p.

6pt b Compute exactly the maximal distance between the points F_p and G_p .

Circle *c* is the circle with diameter *OD*.

6pt c Compute algebraically the distance between point *C* and circle *c*.
Give your answer rounded to 4 digits behind the decimal point.

V is the bounded region enclosed by the *x*-axis, the *y*-axis and the graph of g.

8pt d Compute exactly the volume of the solid of revolution that is formed by rotating *V* around the *y*-axis.

Question 3 – Trigonometric functions

Take a new answer sheet for every question!

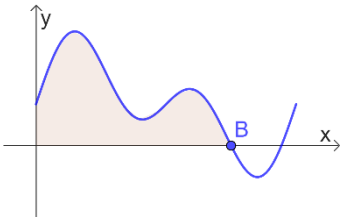
For $0 \leq x \leq 2\pi$, the function f is given by $f(x) = \sin(x) + \sin(2x) + 1$.

4pt a Use an exact computation to show that for all a with $0 \leq a \leq \pi$ we have: $f(\pi + a) + f(\pi - a) = 2$.

Point A is the point on the graph of f with $x_A = \pi$.

5pt b Use an exact computation to show that point A is an inflection point of the graph of f.

In the figure on the right, the graph of f is shown.
Point $B\left(\frac{3}{2}\pi, 0\right)$ is an intersection of this graph and the *x*-axis.
Region *V*, the bounded region enclosed by the graph of f, the *x*-axis and the *y*-axis, is shaded in the figure.



6pt c Compute exactly the area of region *V*.

The function g is given by $g(x) = \sin(x) - \cos(3x) + 1$.

6pt d Compute exactly the *x*-coordinates of the intersections of the graphs of f and g on the interval $0 \leq x \leq 2\pi$.

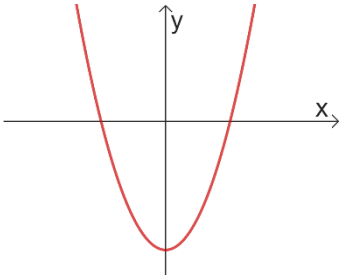
Question 4 – A family of parabolas and a hybrid function

Take a new answer sheet for every question!

The family functions f_a with $a > 0$ is given by

$$f_a(x) = x^2 - a.$$

As an example, the graph of f_4 is shown in the figure on the right.



For each $a > 0$, V_a is the bounded region enclosed by the graph of f_a and the *x*-axis.

7pt a Compute exactly the value of a for which the area of region V_a is equal to 36.

The function g is given by

$$g(x) = \frac{1 - x^2}{1 - e^{x-1}}$$

7pt b Compute exactly the coordinates of the intersections of the graph of g and the graph of $f_1(x) = x^2 - 1$.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t+u) = \sin t \cos u + \cos t \sin u$

$\sin(t-u) = \sin t \cos u - \cos t \sin u$

$\cos(t+u) = \cos t \cos u - \sin t \sin u$

$\cos(t-u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2\sin(t)\cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$

